

Exotic mesons with double charm and bottom flavor

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(Dated: February 6, 2012)

Abstract

We study genuinely exotic mesons with double charm and bottom flavor. We take a hadronic picture by considering the molecular states composed of a pair of heavy mesons, such as DD, DD* and D*D* for charm flavor, and BB, BB* and B*B* for bottom flavor. The interactions between heavy mesons are derived from the heavy quark effective theory. All molecular states are classified by $I(J^P)$ quantum numbers, and are systematically studied up to the total angular momentum $J \leq 2$. For the two-body problem, we numerically solve the coupled channel Schrödinger equations. We find the bound and/or resonant states for many quantum numbers. Some of them are new states which are not predicted by the tetraquark picture.

PACS numbers: 12.39.Jh, 13.30.Eg, 14.20.-c, 12.39.Hg

I. INTRODUCTION

Exotic hadrons which include multiquark configurations provide us with important information in hadron physics. There is a key to understand one of the most important problems in hadron physics; what are the constituent particles of hadrons, and what are the interactions among the constituent particles at relevant low energies. Nowadays the exotic hadrons are studied, not only in light flavor sectors, but also in heavy flavor sectors with charm and bottom quarks [1–5]. Recent experimental observations of heavy exotic hadrons, D_{sJ} , X , Y , Z^\pm in charm sector, and Y_b , Z_b^\pm in bottom sector have motivated many physicists toward the question about the possible new dynamics in the heavy hadrons. Many of those hadrons have masses, decay widths and branching ratios, which may not be explained as normal hadrons, such as $\bar{q}q$ and qqq . Especially Z^\pm and Z_b^\pm which have an isospin one cannot be assigned as simple quarkonia, but there should be additional up and down quark degrees of freedom. Their quark contents will be considered as $\bar{Q}Q\bar{q}q$ with a heavy quark Q and a light quark q , because $\bar{q}q$ can form isospin one.

In the present paper, we consider a new hadron state T_{QQ} whose quark content is $\bar{Q}\bar{Q}qq$ [6–29, 37–39]. T_{QQ} is a system containing two heavy quarks and is genuinely exotic, because there is no annihilation process of a quark and antiquark pair. As discussed in many literatures [6–29], T_{QQ} is considered as a tetraquark state, in which the effective degrees of freedom are constituent quarks. There, it is shown that T_{QQ} can be a stable object due to the strong attraction in qq which may form a stable scalar diquark [30–32]. T_{QQ} may be a deeply bound state, which does not decay through strong interaction [22–24]. The study of T_{QQ} is also useful to understand the color superconductivity in high density quark matter, because the diquark correlations are important to cause a diquark condensate [33–35]. So far tetraquark states such as T_{cc}^1 , T_{cb}^1 and T_{bb}^1 states with $I(J^P) = 0(1^+)$ as well as T_{cb}^0 with $I(J^P) = 0(0^+)$ have been discussed as stable objects. [23, 24].

However, the deeply bound tetraquark is not the only possible picture for T_{QQ} . When four quarks are present, they may form $\bar{Q}q$ hadronic clusters which alternatively become relevant degrees of freedom instead of diquarks. This is the basis of hadronic molecular states which are expected to appear near the hadronic threshold region [37–39]. In the present T_{QQ} system, two mesons composed by $\bar{Q}q$, the pseudoscalar meson $P \sim (\bar{Q}q)_{\text{spin } 0}$ and the vector meson $P^* \sim (\bar{Q}q)_{\text{spin } 1}$, can become effective degrees of freedom as constituents. In the heavy

quark limit, the pseudoscalar meson P and the vector meson P^* become degenerate in mass, and hence both of them should be considered. Hereafter we introduce the notation $P^{(*)}$ for P and P^* .

The interaction between two $P^{(*)}$'s is supplied by the meson exchange interaction. Because $P^{(*)}$ carry a finite isospin, the pion (π) exchange interaction works dominantly at long distances. The existence of a pion is a robust consequence of spontaneous breaking of chiral symmetry [36]. Thus, the mass degeneracy of P and P^* is concerned with the heavy quark (spin) symmetry, and the interaction between two $P^{(*)}$'s with chiral symmetry. Therefore, those two symmetries are important when T_{QQ} is considered as the hadronic molecule of $P^{(*)}P^{(*)}$.

Until now there have been several studies for hadronic bound states of $P^{(*)}P^{(*)}$. The pioneering work by Manohar and Wise showed that the $PP^{(*)}$ mesons could form a bound state [37], and the result has been followed by other authors [38, 39]. However, the quantum numbers $I(J^P)$ which have been discussed so far are limited. Moreover, resonant states of $P^{(*)}P^{(*)}$ have not been studied so far. In the present work, we investigate systematically various states with possible quantum numbers $I(J^P)$ with $J \leq 2$, and discuss both bound and resonant states of $P^{(*)}P^{(*)}$ by considering the fully coupled channel problem.

This paper is organized as follows. In section 2, we give the interaction between two $P^{(*)}$ mesons with respecting the heavy quark symmetry and chiral symmetry. We introduce the two types of potentials, the π exchange potential and $\pi\rho\omega$ exchange potential. In section 3, we apply these potentials to the $P^{(*)}P^{(*)}$ systems and solve the Schrödinger equations numerically to search the bound and/or resonant states. In section 4, we compare our results from the hadronic molecule picture with the previous results from the tetraquark picture. We summarize our discussions in the final section.

II. INTERACTION WITH TWO MESONS WITH DOUBLY HEAVY FLAVOR

The dynamics of the hadronic molecule of $P^{(*)}P^{(*)}$ respects two important symmetries; the heavy quark symmetry and chiral symmetry. The heavy quark symmetry induces the mass degeneracy of P and P^* in the heavy quark limit. Because of this, we have to consider the channels of degenerate pairs, such as PP , PP^* , P^*P and P^*P^* , leading to the mixing of them; PP^*-P^*P , $P^*P^*-P^*P^*$, $PP-P^*P^*$, $PP^*-P^*P^*$.

As for the interaction, the one pion exchange potential (OPEP) exists between $P^{(*)}$'s at long distances. The OPEP is provided by the $PP^*\pi$ and $P^*P^*\pi$ vertices whose coupling strengths are equally weighted thanks to the heavy quark symmetry. We note that there is no $PP\pi$ vertex because of the parity conservation. Therefore, at long distances, the OPEP plays an important role to realize the channel mixings in $P^{(*)}P^{(*)}$. At short distances, heavier mesons also should be considered.

To derive the $P^{(*)}P^{(*)}$ potential, we employ the effective Lagrangians based on the heavy quark symmetry and chiral symmetry [40–45]. They describe the interaction between heavy mesons which are given by the exchange of pion and vector mesons ($v = \rho, \omega$). The interaction Lagrangians are given as

$$\mathcal{L}_{\pi PP^*} = 2\frac{g}{f_\pi}(P_a^\dagger P_{b\mu}^* + P_{a\mu}^{*\dagger} P_b)\partial^\mu \hat{\pi}_{ab}, \quad (1)$$

$$\mathcal{L}_{\pi P^*P^*} = 2i\frac{g}{f_\pi}\epsilon^{\alpha\beta\mu\nu}v_\alpha P_{a\beta}^{*\dagger} P_{b\mu}^* \partial_\nu \hat{\pi}_{ab}, \quad (2)$$

$$\mathcal{L}_{v PP} = -\sqrt{2}\beta g_V P_b P_a^\dagger v \cdot \hat{\rho}_{ba}, \quad (3)$$

$$\mathcal{L}_{v PP^*} = -2\sqrt{2}\lambda g_V v_\mu \epsilon^{\mu\nu\alpha\beta} \left(P_a^\dagger P_{b\beta}^* - P_{a\beta}^{*\dagger} P_b \right) \partial_\nu (\hat{\rho}_\alpha)_{ba}, \quad (4)$$

$$\begin{aligned} \mathcal{L}_{v P^*P^*} = & \sqrt{2}\beta g_V P_b^* P_{a\mu}^{*\dagger} v \cdot \hat{\rho}_{ba} \\ & + i2\sqrt{2}\lambda g_V P_{a\mu}^{*\dagger} P_{b\nu}^* (\partial^\mu (\hat{\rho}^\nu)_{ba} - \partial^\nu (\hat{\rho}^\mu)_{ba}), \end{aligned} \quad (5)$$

where $P = D, B$ and $P_\mu^* = D_\mu^*, B_\mu^*$. The subscripts a and b are for light flavor indices, up and down, and v_μ is a four-velocity which will be fixed as $v_\mu = (1, \vec{0})$ below. The pion and vector meson (ρ and ω) fields are defined by

$$\hat{\pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}, \quad \hat{\rho}_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}_\mu. \quad (6)$$

Following Ref. [45], the coupling constants in the interaction Lagrangians are given as

$$g = 0.59, \quad \beta = 0.9, \quad \lambda = 0.56 \text{ GeV}^{-1}, \quad f_\pi = 132 \text{ MeV}, \quad g_V = \frac{m_\rho}{f_\pi}, \quad (7)$$

where f_π is the pion decay constant and m_ρ is the mass of the ρ meson. The OPEPs are

derived by the interaction Lagrangians (1) and (2) as follows:

$$V_{P_1 P_2^* \rightarrow P_1^* P_2}^\pi = \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} [\vec{\varepsilon}_1^* \cdot \vec{\varepsilon}_2 C(r; m_\pi) + S_{\varepsilon_1^*, \varepsilon_2} T(r; m_\pi)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (8)$$

$$V_{P_1^* P_2^* \rightarrow P_1^* P_2^*}^\pi = \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} [\vec{T}_1 \cdot \vec{T}_2 C(r; m_\pi) + S_{T_1, T_2} T(r; m_\pi)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (9)$$

$$V_{P_1 P_2 \rightarrow P_1^* P_2^*}^\pi = \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} [\vec{\varepsilon}_1^* \cdot \vec{\varepsilon}_2^* C(r; m_\pi) + S_{\varepsilon_1^*, \varepsilon_2^*} T(r; m_\pi)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (10)$$

$$V_{P_1 P_2^* \rightarrow P_1^* P_2^*}^\pi = - \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} [\vec{\varepsilon}_1^* \cdot \vec{T}_2 C(r; m_\pi) + S_{\varepsilon_1^*, T_2} T(r; m_\pi)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (11)$$

where m_π is the pion mass. Here three polarizations are introduced for P^* as defined by $\vec{\varepsilon}^{(\pm)} = (\mp 1/\sqrt{2}, \pm i/\sqrt{2}, 0)$ and $\vec{\varepsilon}^{(0)} = (0, 0, 1)$, and the spin-one operator \vec{T} is defined by $T_{\lambda'\lambda}^i = i\varepsilon^{ijk}\varepsilon_j^{(\lambda')\dagger}\varepsilon_k^{(\lambda)}$. As a convention, we assign $\vec{\varepsilon}^{(\lambda)}$ for an incoming vector particle and $\vec{\varepsilon}^{(\lambda)*}$ for an outgoing vector particle. Here $\vec{\tau}_1$ and $\vec{\tau}_2$ are isospin operators for $P_1^{(*)}$ and $P_2^{(*)}$; $\vec{\tau}_1 \cdot \vec{\tau}_2 = -3$ and 1 for the $I = 0$ and $I = 1$ channels, respectively. We define the tensor operators

$$S_{\varepsilon_1^*, \varepsilon_2} = 3(\vec{\varepsilon}^{(\lambda_1)*} \cdot \hat{r})(\vec{\varepsilon}^{(\lambda_2)} \cdot \hat{r}) - \vec{\varepsilon}^{(\lambda_1)*} \cdot \vec{\varepsilon}^{(\lambda_2)}, \quad (12)$$

$$S_{T_1, T_2} = 3(\vec{T}_1 \cdot \hat{r})(\vec{T}_2 \cdot \hat{r}) - \vec{T}_1 \cdot \vec{T}_2, \quad (13)$$

$$S_{\varepsilon_1^*, \varepsilon_2^*} = 3(\vec{\varepsilon}^{(\lambda_1)*} \cdot \hat{r})(\vec{\varepsilon}^{(\lambda_2)*} \cdot \hat{r}) - \vec{\varepsilon}^{(\lambda_1)*} \cdot \vec{\varepsilon}^{(\lambda_2)*}, \quad (14)$$

$$S_{\varepsilon_1^*, T_2} = 3(\vec{\varepsilon}^{(\lambda_1)*} \cdot \hat{r})(\vec{T}_2 \cdot \hat{r}) - \vec{\varepsilon}^{(\lambda_1)*} \cdot \vec{T}_2, \quad (15)$$

where $\hat{r} = \vec{r}/r$ is a unit vector between the two mesons.

The ρ meson exchange potentials are similarly obtained from the interaction Lagrangians (3)-(5),

$$V_{P_1 P_2 \rightarrow P_1 P_2}^\rho = \left(\frac{\beta g_V}{2m_\rho} \right)^2 \frac{1}{3} C(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (16)$$

$$V_{P_1 P_2^* \rightarrow P_1 P_2^*}^\rho = \left(\frac{\beta g_V}{2m_\rho} \right)^2 \frac{1}{3} C(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (17)$$

$$V_{P_1 P_2^* \rightarrow P_1^* P_2}^\rho = (2\lambda g_V)^2 \frac{1}{3} [2\vec{\varepsilon}_1^* \cdot \vec{\varepsilon}_2 C(r; m_\rho) - S_{\varepsilon_1^*, \varepsilon_2} T(r; m_\rho)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (18)$$

$$V_{P_1^* P_2^* \rightarrow P_1^* P_2^*}^\rho = (2\lambda g_V)^2 \frac{1}{3} [2\vec{T}_1 \cdot \vec{T}_2 C(r; m_\rho) - S_{T_1, T_2} T(r; m_\rho)] \vec{\tau}_1 \cdot \vec{\tau}_2 \\ + \left(\frac{\beta g_V}{2m_\rho} \right)^2 \frac{1}{3} C(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (19)$$

$$V_{P_1 P_2 \rightarrow P_1^* P_2^*}^\rho = (2\lambda g_V)^2 \frac{1}{3} [2\vec{\varepsilon}_1^* \cdot \vec{\varepsilon}_2^* C(r; m_\rho) - S_{\varepsilon_1^*, \varepsilon_2^*} T(r; m_\rho)] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (20)$$

$$V_{P_1 P_2^* \rightarrow P_1^* P_2^*}^\rho = - (2\lambda g_V)^2 \frac{1}{3} [2\vec{\varepsilon}_1^* \cdot \vec{T}_2 C(r; m_\rho) - S_{\varepsilon_1^*, T_2} T(r; m_\rho)] \vec{\tau}_1 \cdot \vec{\tau}_2. \quad (21)$$

The ω meson exchange potentials are obtained by replacing the mass of ρ meson with the one of ω meson and by removing the isospin factor $\vec{\tau}_1 \cdot \vec{\tau}_2$. The OPEP's of $P^{(*)}P^{(*)}$ differ from the ones of $P^{(*)}\bar{P}^{(*)}$ in that the overall signs are changed due to G -parity. The situation is the same with ω meson exchange potentials, while ρ meson exchange potentials of $P^{(*)}\bar{P}^{(*)}$ are not changed because the G -parity is even [48].

In the above equations, $C(r; m_h)$ and $T(r; m_h)$ are defined as

$$C(r; m_h) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{m_h^2}{\vec{q}^2 + m_h^2} e^{i\vec{q} \cdot \vec{r}} F(\vec{q}; m_h), \quad (22)$$

$$T(r; m_h) S_{12}(\hat{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{-\vec{q}^2}{\vec{q}^2 + m_h^2} S_{12}(\hat{q}) e^{i\vec{q} \cdot \vec{r}} F(\vec{q}; m_h), \quad (23)$$

with $S_{12}(\hat{x}) = 3(\vec{\sigma}_1 \cdot \hat{x})(\vec{\sigma}_2 \cdot \hat{x}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$. We introduce the monopole type form factor at each vertex to take into account of the size effect of $P^{(*)}$ mesons. So the function reflected form factors is defined as

$$F(\vec{q}; m_h) = \left(\frac{\Lambda_P^2 - m_h^2}{\Lambda_P^2 + \vec{q}^2} \right)^2, \quad (24)$$

where m_h and \vec{q} are the mass and three-momentum of the exchanged meson h ($= \pi, \rho, \omega$) and Λ_P is the cut-off parameter. The cut-off parameter Λ_P are determined from the size of P estimated from the constituent quark model as discussed in Refs. [46–49]. The cut-off parameters are $\Lambda_D = 1121$ MeV and $\Lambda_B = 1070$ MeV when the π exchange potential is employed, while $\Lambda_D = 1142$ MeV and $\Lambda_B = 1091$ MeV when the $\pi\rho\omega$ is employed.

III. BOUND AND RESONANT STATES

Let us classify all the possible quantum numbers of the $P^{(*)}P^{(*)}$ systems with isospin I , total angular momentum J ($J \leq 2$), and parity P . We show the quantum numbers $I(J^P)$ and the channels in the wave functions in Table I. It is noted that the wave functions must be symmetric under the exchange of the two $P^{(*)}$ mesons. We use the notation $^{2S+1}L_J$ to indicate the states with the internal spins S and angular momentum L . For example, the $I(J^P) = 0(1^+)$ state is a superposition of four channels; $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3S_1)$, $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3D_1)$, $P^*P(^3S_1)$ and $P^*P(^3D_1)$. It is important to consider the mixing of all possible channels. Especially the tensor force plays a crucial role to mix the channels with different angular momenta, L and $L \pm 2$. We obtain the Hamiltonian in a matrix

form of these coupled channels. The explicit forms of the Hamiltonian are summarized in Appendix A.

We numerically solve the coupled channel Schrödinger equations for the states with each quantum number. As a numerical method, the renormalized Numerov method [50] is adopted to solve the coupled second-order differential equations. The resonant states are identified by the behavior of the phase shift δ as a function of the scattering energy E . The resonance position E_r is defined by an inflection point of the phase shift $\delta(E)$ and the resonance width by $\Gamma_r = 2/(d\delta/dE)_{E=E_r}$ following Ref. [47, 48, 51]. To check the consistency of our numerical calculations, we also adopt the complex scaling method (CSM), in which the resonant state is defined as a pole in the complex energy plane [51, 52]. We obtain an agreement in the results of the renormalized Numerov method and the CSM.

We summarize our numerical results for $D^{(*)}D^{(*)}$ bound/resonant states in Table II and Fig 1. In $D^{(*)}D^{(*)}$ states, we find several bound and/or resonant states in $I = 0$, while there is no bound state in $I = 1$. In general, the attractive force of pion exchange in $I = 1$ is three times weaker than in $I = 0$ due to the isospin factor. As a numerical result, $D^{(*)}D^{(*)}$ bound states in $I = 1$ are not obtained but only resonant states are.

Let us look at our results one by one for each quantum number in detail. In the followings, most of the numerical values are shown for the case of the $\pi\rho\omega$ potential, because the results from the $\pi\rho\omega$ potential are generally not so different from those from the π potential, except for the $0(2^-)$ state.

$0(0^-)$ This state has only one channel and the pion exchange potential is attractive as shown in Eq. (A14). As a result, this state is a very deeply bound state with binding energy 132.1 MeV measured from the DD^* threshold.

$0(1^+)$ The pion exchange potential is repulsive for diagonal components as shown in Eq. (A15). However this state has four components and the mixing of the S - and D -waves causes the strong tensor attraction from the off-diagonal components of the potential. Consequently, there is a deeply bound state with binding energy 62.3 MeV measured from the DD^* threshold.

$0(1^-)$ There are twin shape resonances with the resonance energy 17.8 MeV and the decay width 41.6 MeV for the first resonance, and the resonance energy 152.8 MeV and the

decay width 10.6 MeV for the second. The resonance energies are measured from the DD threshold. Those resonances are formed by the centrifugal barrier in the P -wave.

$0(2^+)$ This state contains only D -wave components. The potential is weakly attractive. Nevertheless, due to the centrifugal barrier in the D -wave, there is a resonant state at the energy 33.7 MeV from the DD* threshold, but the decay width 196.3 MeV is very wide.

$0(2^-)$ When the OPEP is employed, there are twin resonant states with the resonance energy 0.1 MeV and the decay width 0.02 MeV for the first resonance, and the resonance energy 118.0 MeV and the decay width 23.4 MeV for the second from the DD* threshold. When the effects of ρ and ω meson exchange are included, the first resonance becomes a weakly bound state with the binding energy 4.3 MeV, because the ρ meson exchange enhances the central force attraction of the pion exchange. The ω meson exchange plays a minor role due to the isospin factor, although this contribution suppresses the attractive central force. The second resonant state with the resonance energy 112.1 MeV and the decay width 26.6 MeV is not affected very much.

$1(0^-)$ This is the only $I = 1$ state in $D^{(*)}D^{(*)}$. The interaction in $I = 1$ are either repulsive or only weakly attractive as already discussed. Nevertheless, due to the P -wave centrifugal barrier, we find twin shape resonances; the resonance energy 2.3 MeV and the decay width 37.4 MeV for the first resonance, and the resonance energy 144.2 MeV and the decay width 34.4 MeV for the second, from the DD* threshold.

Next we discuss the results for the $B^{(*)}B^{(*)}$ states in Table III and Figs. 2 and 3. In Figs. 2 and 3, only states around the threshold region are plotted. For $B^{(*)}B^{(*)}$ states, we use the same coupling constants, and change only the masses of heavy mesons with small difference of the cutoff parameters. At first glance $B^{(*)}B^{(*)}$ states have many bound and resonant states in comparison with $D^{(*)}D^{(*)}$ states. There are two reasons. First, the kinetic term is suppressed in the Hamiltonian because the reduced mass becomes larger in bottom sector. Second, the effect of channel-couplings becomes more important in the bottom sector, because a pseudoscalar meson B and a vector meson B^* become more degenerate thanks to the heavy quark symmetry; similar discussion has been done in Refs. [46–48]. We note that, as a consequence of the strong attraction, several new states appear in the $B^{(*)}B^{(*)}$ states

in isospin triplet; $I(J^P) = 1(0^+)$, $1(1^-)$, $1(2^+)$ and $1(2^-)$. The corresponding states are not obtained in the $D^{(*)}D^{(*)}$ states.

Here we comment on the similarities in the energy spectrum in the charm and bottom sectors. When we look at the $I = 0$ states in Figs. 1 and 2, and the $1(0^-)$ states in Figs. 1 and 3, we find qualitatively the correspondence between the states in charm and bottom sectors. For instance, $I(J^P) = 0(0^-)$ has one state, and $I(J^P) = 0(1^-)$ has two states in both charm and bottom sectors. The correspondence between the two sectors may lead us to the idea that the energy spectrum around the thresholds does not depend on the heavy flavor very much, when the PP , PP^* and P^*P^* thresholds are close each other. If we look at their levels in the threshold region in more detail in $I = 0$, the states in the charm sector are those of principal quantum number $n = 1$ (lowest) or $n = 2$, while those in the bottom sector plotted in Figs 2 and 3 are with $n = 3$. The correspondence is more clearly seen in the $1(0^-)$ states in $I = 1$, where these states happen to be shallow bound states or resonant states both in charm and bottom sectors. Therefore, we observe that the states appearing around the thresholds exist as hadronic molecules and share similarity in their spectrum. The states with stronger binding far below the thresholds with smaller n may no longer survive as hadronic molecules, but rather tetraquarks, or mixtures of hadronic molecules and tetraquarks [54]. The tetraquark will be briefly discussed in the next section.

IV. HADRONIC MOLECULES OR TETRAQUARKS?

In the present paper, we have considered T_{QQ} as a hadronic molecule composed by $P^{(*)}P^{(*)}$ in which one pion exchange potential induces a dominant attraction. On the other hand, as mentioned in the introduction, T_{QQ} can also be considered as a tetraquark $\bar{Q}\bar{Q}qq$ in which a diquark qq provides a strong binding energy. We briefly comment on the difference of those two pictures.

The different features between the hadronic molecule and tetraquark pictures are seen in their masses. For hadronic molecules to be valid, hadron constituents are sufficiently far apart such that their identities as hadrons must be maintained. They can not be too close to overlap each other. Therefore, masses of hadronic molecules should appear around their threshold regions. In contrast, tetraquarks may be strongly bound and become compact objects as genuine quark objects. Thus, their natures are differentiated in the masses, small

TABLE I: Several channels of $P^{(*)}P^{(*)}(2S+1L_J)$ in each state with I and J^P for $J \leq 2$.

| I | J^P | components |
|-----|-------|---|
| 0 | 0^- | $\frac{1}{\sqrt{2}}(PP^* + P^*P)(^3P_0)$ |
| | 1^+ | $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3S_1), \frac{1}{\sqrt{2}}(PP^* - P^*P)(^3D_1), P^*P^*(^3S_1), P^*P^*(^3D_1)$ |
| | 1^- | $PP(^1P_1), \frac{1}{\sqrt{2}}(PP^* + P^*P)(^3P_1), P^*P^*(^1P_1), P^*P^*(^5P_1), P^*P^*(^5F_1)$ |
| | 2^+ | $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3D_2), P^*P^*(^3D_2)$ |
| | 2^- | $\frac{1}{\sqrt{2}}(PP^* + P^*P)(^3P_2), \frac{1}{\sqrt{2}}(PP^* + P^*P)(^3F_2), P^*P^*(^5P_2), P^*P^*(^5F_2)$ |
| 1 | 0^+ | $PP(^1S_0), P^*P^*(^1S_0), P^*P^*(^5D_0)$ |
| | 0^- | $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3P_0), P^*P^*(^3P_0)$ |
| | 1^+ | $\frac{1}{\sqrt{2}}(PP^* + P^*P)(^3S_1), \frac{1}{\sqrt{2}}(PP^* + P^*P)(^3D_1), P^*P^*(^5D_1)$ |
| | 1^- | $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3P_1), P^*P^*(^3P_1)$ |
| | 2^+ | $PP(^1D_2), \frac{1}{\sqrt{2}}(PP^* + P^*P)(^3D_2), P^*P^*(^1D_2), P^*P^*(^5S_2), P^*P^*(^5D_2), P^*P^*(^5G_2)$ |
| | 2^- | $\frac{1}{\sqrt{2}}(PP^* - P^*P)(^3P_2), \frac{1}{\sqrt{2}}(PP^* - P^*P)(^3F_2), P^*P^*(^3P_2), P^*P^*(^3F_2)$ |

binding energy of order ten MeV or less, or larger one. Although it is tempting to seek for a framework to cover both scales, such a problem is out of scope of the present paper. Instead, we compare the results from the two pictures and just clarify the differences between them.

In the hadronic molecule picture by $P^{(*)}P^{(*)}$, as presented in the previous section, we obtain, not only the bound states, but also the resonant states in many $I(J^P)$ quantum numbers. Both in charm and bottom sectors, it is remarkable that there are even the twin resonances in several quantum numbers, $0(1^-)$, $0(2^-)$ and $1(0^-)$. In the tetraquark picture including a diquark model [23, 24], in contrast, only two bound states in $I(J^P) = 0(1^+)$ and $1(2^-)$ have been predicted so far, as discussed for an example in Refs. [27–29] as recent works. For $0(1^+)$, the predicted binding energy can be around 70 MeV from the DD^* threshold,

TABLE II: The energies of $D^{(*)}D^{(*)}$ states with $I(J^P)$ with $J \leq 2$. The energies E can be either pure real for bound states or complex for resonances. The real parts are measured from the thresholds as indicated in the second columns. The imaginary parts are half of the decay widths of the resonances, $\Gamma/2$. The values in the parentheses for the bound states are matter radii in units of fm.

| I | J^P | threshold | E [MeV] | |
|-----|-------|-----------|-------------------------------------|-----------------------------------|
| | | | π -potential | π, ρ, ω -potential |
| 0 | 0^- | DD^* | $-50.2(1.0)$ | $-132.1(0.8)$ |
| | 1^+ | DD^* | $-45.7(0.9)$ | $-62.3(0.8)$ |
| | 1^- | DD | $19.4 - i63.0/2/2, 175.4 - i37.4/2$ | $17.8 - i41.6/2, 152.8 - i10.6/2$ |
| | 2^+ | DD^* | $34.5 - i183.1/2$ | $33.7 - i196.3/2$ |
| | 2^- | DD^* | $0.1 - i0.02/2, 118.0 - i23.4/2$ | $-4.3(1.6), 112.1 - i26.6/2$ |
| 1 | 0^+ | DD | no | no |
| | 0^- | DD^* | $3.7 - i27.7/2, 143.8 - i40.0/2$ | $2.3 - i37.4/2, 144.2 - i34.4/2$ |
| | 1^+ | DD^* | no | no |
| | 1^- | DD^* | no | no |
| | 2^+ | DD | $289.4 - i10.9/2$ | no |
| | 2^- | DD^* | no | no |

which is accidentally similar to our values 62 MeV. For $1(2^-)$, the predicted binding energy can be around 27 MeV. The $1(2^-)$ state cannot be found in our present study. Thus, we observe that the energy spectrum of the hadronic molecule picture by $P^{(*)}P^{(*)}$ is quite different from that of the tetraquark picture.

TABLE III: The energies of $B^{(*)}B^{(*)}$ states with $I(J^P)$ with $J \leq 2$. (Same convention as Table II.)

| I | J^P | threshold | $E - i\Gamma/2$ [MeV] | |
|-----|-------|-----------|---|--|
| | | | π -potential | π, ρ, ω -potential |
| 0 | 0^- | BB^* | 178.0(0.6), $-32.0(1.2)$ | $-305.9(0.5)$, $-77.5(0.9)$, $-3.3(2.5)$ |
| | 1^+ | BB^* | $-179.7(0.5)$, $-25.7(1.2)$ | $-201.5(0.5)$, $-33.6(1.1)$ |
| | 1^- | BB | $-125.5(0.6)$, $-39.1(0.8)$, $1.9 - i3.3/2$, $52.8 - i12.8/2$ | $-164.4(0.5)$, $-98.9(0.6)$, $-3.1(1.6)$, $35.0 - i11.8/2$ |
| | 2^+ | BB^* | $-51.2(0.8)$, $5.5 - i5.9/2$ | $-60.6(0.8)$, $5.7 - i13.2/2$ |
| | 2^- | BB^* | $-147.5(0.6)$, $-68.7(0.8)$, $7.6 - i4.3/2$, $26.9 - i20.2/2$ | $-196.5(0.6)$, $-84.1(0.7)$ $0.5 - i8.5/2$, $7.6 - i4.3/2$, $26.9 - i20.2/2$ |
| | | | | |
| 1 | 0^+ | BB | $-18.1(0.9)$, | $-33.9(0.5)$ |
| | 0^- | BB^* | $-50.5(0.8)$, $0.7 - i3.0/2$, $46.7 - i1.9/2$ | $-5.9(1.4)$, $38.5 - i4.6/2$ |
| | 1^+ | BB^* | $-38.1(0.8)$ | no |
| | 1^- | BB^* | no | $11.7 - i11.0/2$ |
| | 2^+ | BB | $23.0 - i4.4/2$ | $62.4 - i52.5/2$ |
| | 2^- | BB^* | $2.0 - i3.3/2$, $63.7 - i7.6/2$ | $2.3 - i4.7/2$ |

V. SUMMARY

We have discussed exotic mesons with double charm and bottom flavor whose quark content $\bar{Q}\bar{Q}qq$ is genuinely exotic. We have taken the hadronic picture, and considered molecular states of two heavy mesons $P^{(*)}$'s (a pseudoscalar meson $P = D, B$, and a vector meson $P^* = D^*, B^*$). With respecting the heavy quark and chiral symmetries, we have constructed the π exchange potential and the $\pi\rho\omega$ exchange potential between the two heavy mesons. To investigate the bound and/or resonant states, we have numerically solved the coupled channel Schrödinger equations for the $P^{(*)}P^{(*)}$ states with $I(J^P)$ for $J \leq 2$.

As results, we have found many bound and/or resonant states in both charm and bottom

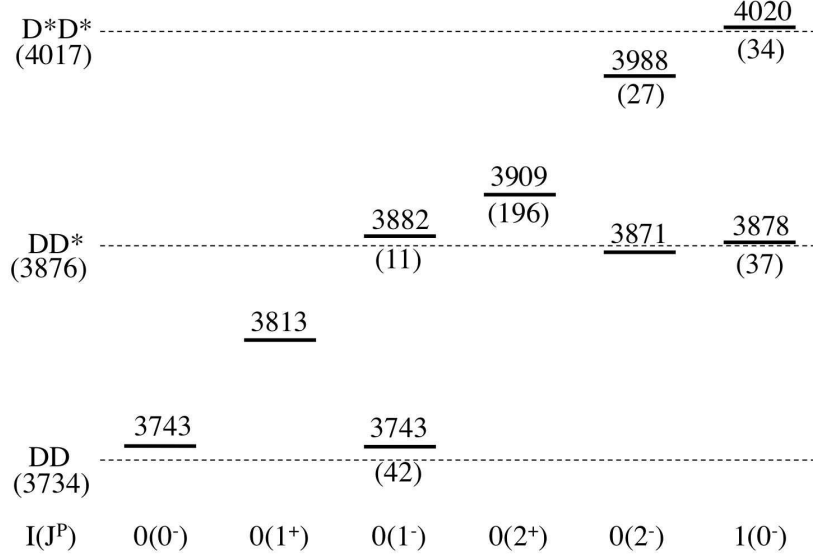


FIG. 1: The $D^{(*)}D^{(*)}$ bound and resonant states with $I(J^P)$. Solid lines are for our predictions for the energies E of the bound and resonant states, as denoted above the lines, and the values in parentheses below the lines denote the decay width Γ of the resonances when the $\pi\rho\omega$ potential is employed. Mass values are shown in units of MeV.

sectors. The $D^{(*)}D^{(*)}$ bound and resonant states have moderate energies and decay widths around the thresholds in several channels with quantum numbers; $0(0^-)$, $0(1^+)$, $0(1^-)$, $0(2^+)$, $0(2^-)$ and $1(0^-)$. The $B^{(*)}B^{(*)}$ states have more bound and resonant states with various quantum numbers. Several new states appear in the $B^{(*)}B^{(*)}$ states in isotriplet states, such as $1(0^+)$, $1(1^-)$, $1(2^+)$ and $1(2^-)$, which cannot be found in the charm sector. By contrast to the $D^{(*)}D^{(*)}$ states, some $B^{(*)}B^{(*)}$ states are very compact objects with a large binding energy much below the thresholds. Perhaps, these states cannot survive as hadronic molecules and more consideration of quark dynamics such as tetraquarks is required.

Experimental studies of those exotic hadrons should be performed in the coming future. The double charm production in accelerator facilities will help us to search them [53]. Recently it has been discussed that the quark-gluon plasma in the relativistic heavy ion collisions could produce much abundance of exotic hadrons including the exotic mesons with double charm [55, 56]. Those experimental studies will shed a light on the nature of the

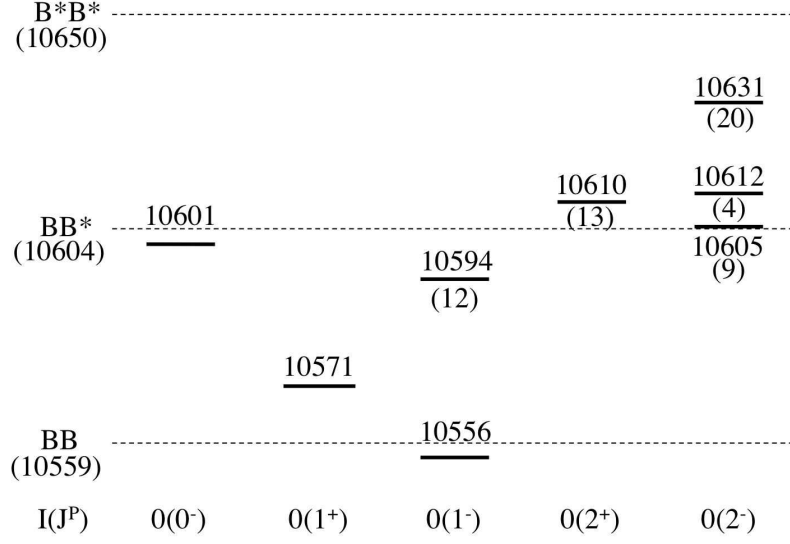


FIG. 2: The $B^{(*)}B^{(*)}$ bound and resonant states around the thresholds with $I(J^P)$ in $I = 0$. (Same convention as Fig. 1.)

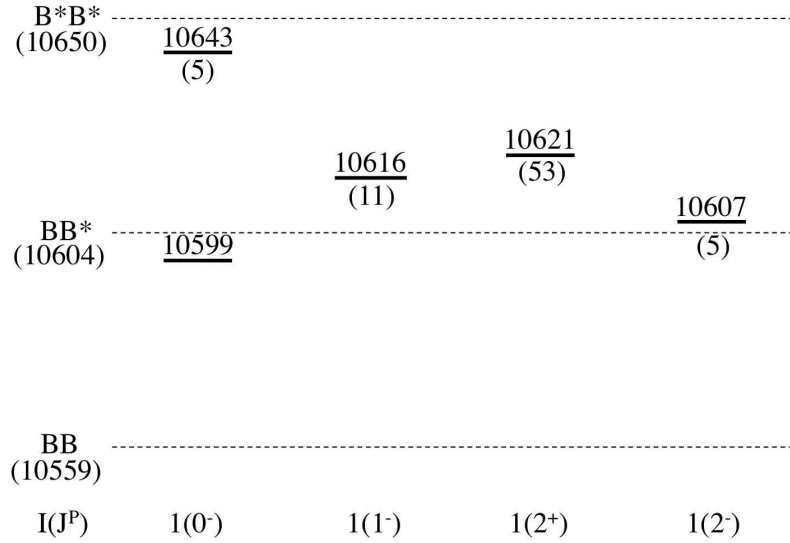


FIG. 3: The $B^{(*)}B^{(*)}$ bound and resonant states around the thresholds with $I(J^P)$ in $I = 1$. (Same convention as Fig. 1.)

exotic mesons with double charm and bottom flavor, and provide hints to the fundamental questions of the strong interaction in hadron physics.

Acknowledgments

We thank Prof. S. Takeuchi and Prof. M. Takizawa for fruitful discussions and comments. This work is supported in part by Grant-in-Aid for Scientific Research on Priority Areas “Elucidation of New Hadrons with a Variety of Flavors (E01: 21105006)” (S.Y. and A.H.) and by “Grant-in-Aid for Young Scientists (B) 22740174” (K.S.), from the ministry of Education, Culture, Sports, Science and Technology of Japan.

Appendix A: Hamiltonian

The Hamiltonian is a sum of the kinetic term and potential term as,

$$H_{I(J^P)} = K_{I(J^P)} + V_{I(J^P)}^\pi, \quad (\text{A1})$$

for the π exchange potential only, and

$$H_{I(J^P)} = K_{I(J^P)} + \sum_{i=\pi,\rho,\omega} V_{I(J^P)}^i, \quad (\text{A2})$$

for the $\pi\rho\omega$ potential.

The kinetic terms with including the explicit breaking of the heavy quark symmetry by

the mass difference $\Delta m_{\text{PP}^*} = m_{\text{P}^*} - m_{\text{P}}$ are

$$K_{0(0-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1 \right), \quad (\text{A3})$$

$$K_{0(1+)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_0, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_2, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_0 + \Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_2 + \Delta m_{\text{PP}^*} \right), \quad (\text{A4})$$

$$K_{0(1-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}}} \Delta_0, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1 + \Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_1 + 2\Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_3 + 2\Delta m_{\text{PP}^*} \right), \quad (\text{A5})$$

$$K_{0(2+)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_2, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_2 + \Delta m_{\text{PP}^*} \right), \quad (\text{A6})$$

$$K_{0(2-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_3, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_1 + \Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_3 + \Delta m_{\text{PP}^*} \right), \quad (\text{A7})$$

$$K_{1(0+)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}}} \Delta_0, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_0 + 2\Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_2 + 2\Delta m_{\text{PP}^*} \right), \quad (\text{A8})$$

$$K_{1(0-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_1 + \Delta m_{\text{PP}^*} \right), \quad (\text{A9})$$

$$K_{1(1+)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_0, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_2, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_2 + \Delta m_{\text{PP}^*} \right), \quad (\text{A10})$$

$$K_{1(1-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_1 + \Delta m_{\text{PP}^*} \right), \quad (\text{A11})$$

$$K_{1(2+)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}}} \Delta_2, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_2 + \Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_2 + 2\Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_0 + 2\Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_2 + 2\Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_4 + 2\Delta m_{\text{PP}^*} \right), \quad (\text{A12})$$

$$K_{1(2-)} = \text{diag} \left(-\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_1, -\frac{1}{2\tilde{m}_{\text{PP}^*}} \Delta_3, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_1 + \Delta m_{\text{PP}^*}, -\frac{1}{2\tilde{m}_{\text{P}^*\text{P}^*}} \Delta_3 + \Delta m_{\text{PP}^*} \right), \quad (\text{A13})$$

where $\Delta_l = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2}$ with integer $l \geq 0$, $1/\tilde{m}_{\text{PP}} = 1/m_{\text{P}} + 1/m_{\text{P}}$, $1/\tilde{m}_{\text{PP}^*} = 1/m_{\text{P}} + 1/m_{\text{P}^*}$, $1/\tilde{m}_{\text{P}^*\text{P}^*} = 1/m_{\text{P}^*} + 1/m_{\text{P}^*}$.

The π exchange potentials for each $I(J^P)$ states are

$$V_{0(0-)}^\pi = (V_C + 2V_T), \quad (\text{A14})$$

$$V_{0(1+)}^\pi = \begin{pmatrix} -V_C & \sqrt{2}V_T & 2V_C & \sqrt{2}V_T \\ \sqrt{2}V_T & -V_C - V_T & \sqrt{2}V_T & 2V_C - V_T \\ 2V_C & \sqrt{2}V_T & -V_C & \sqrt{2}V_T \\ \sqrt{2}V_T & 2V_C - V_T & \sqrt{2}V_T & -V_C - V_T \end{pmatrix}, \quad (\text{A15})$$

$$V_{0(1-)}^\pi = \begin{pmatrix} 0 & 0 & -\sqrt{3}V_C & -2\sqrt{\frac{3}{5}}V_T & 3\sqrt{\frac{2}{5}}V_T \\ 0 & V_C - V_T & 0 & -3\sqrt{\frac{3}{5}}V_T & -3\sqrt{\frac{2}{5}}V_T \\ -\sqrt{3}V_C & 0 & -2V_C & \frac{2}{\sqrt{5}}V_T & -\sqrt{\frac{6}{5}}V_T \\ -2\sqrt{\frac{3}{5}}V_T & -3\sqrt{\frac{3}{5}}V_T & \frac{2}{\sqrt{5}}V_T & V_C - \frac{7}{5}V_T & \frac{\sqrt{6}}{5}V_T \\ 3\sqrt{\frac{2}{5}}V_T & -3\sqrt{\frac{2}{5}}V_T & -\sqrt{\frac{6}{5}}V_T & \frac{\sqrt{6}}{5}V_T & V_C - \frac{8}{5}V_T \end{pmatrix}, \quad (\text{A16})$$

$$V_{0(2+)}^\pi = \begin{pmatrix} -V_C + V_T & 2V_C + V_T \\ 2V_C + V_T & -V_C + V_T \end{pmatrix}, \quad (\text{A17})$$

$$V_{0(2-)}^\pi = \begin{pmatrix} V_C + \frac{1}{5}V_T & -\frac{3\sqrt{6}}{5}V_T & \frac{3\sqrt{3}}{5}V_T & -\frac{6\sqrt{3}}{5}V_T \\ -\frac{3\sqrt{6}}{5}V_T & V_C + \frac{4}{5}V_T & \frac{3\sqrt{2}}{5}V_T & -\frac{6\sqrt{2}}{5}V_T \\ \frac{3\sqrt{3}}{5}V_T & \frac{3\sqrt{2}}{5}V_T & V_C + \frac{7}{5}V_T & \frac{6}{5}V_T \\ -\frac{6\sqrt{3}}{5}V_T & -\frac{6\sqrt{2}}{5}V_T & \frac{6}{5}V_T & V_C - \frac{2}{5}V_T \end{pmatrix}. \quad (\text{A18})$$

$$V_{1(0+)}^\pi = \begin{pmatrix} 0 & -\sqrt{3}V_C & \sqrt{6}V_T \\ -\sqrt{3}V_C & -2V_C & -\sqrt{2}V_T \\ \sqrt{6}V_T & -\sqrt{2}V_T & V_C - 2V_T \end{pmatrix}, \quad (\text{A19})$$

$$V_{1(0-)}^\pi = \begin{pmatrix} -V_C - 2V_T & 2V_C - 2V_T \\ 2V_C - 2V_T & -V_C - 2V_T \end{pmatrix}, \quad (\text{A20})$$

$$V_{1(1+)}^\pi = \begin{pmatrix} V_C & -\sqrt{2}V_T & -\sqrt{6}V_T \\ -\sqrt{2}V_T & V_C + V_T & -\sqrt{3}V_T \\ -\sqrt{6}V_T & -\sqrt{3}V_T & V_C - V_T \end{pmatrix}, \quad (\text{A21})$$

$$V_{1(1-)}^\pi = \begin{pmatrix} -V_C + V_T & -2V_C - V_T \\ -2V_C - V_T & -V_C + V_T \end{pmatrix}, \quad (\text{A22})$$

$$V_{1(2+)}^\pi = \begin{pmatrix} 0 & 0 & -\sqrt{3}V_C & \sqrt{\frac{6}{5}}V_T & -2\sqrt{\frac{3}{7}}V_T & 6\sqrt{\frac{3}{35}}V_T \\ 0 & V_C - V_T & 0 & 3\sqrt{\frac{2}{5}}V_T & -\frac{3}{\sqrt{7}}V_T & -\frac{12}{\sqrt{35}}V_T \\ -\sqrt{3}V_C & 0 & -2V_C & -\sqrt{\frac{2}{5}}V_T & \frac{2}{\sqrt{7}}V_T & -\frac{6}{\sqrt{35}}V_T \\ \sqrt{\frac{6}{5}}V_T & 3\sqrt{\frac{3}{5}}V_T & -\sqrt{\frac{2}{5}}V_T & V_C & \sqrt{\frac{14}{5}}V_T & 0 \\ -2\sqrt{\frac{3}{7}}V_T & -\frac{3}{\sqrt{7}}V_T & \frac{2}{\sqrt{7}}V_T & \sqrt{\frac{14}{5}}V_T & V_C + \frac{3}{7}V_T & \frac{12}{7\sqrt{5}}V_T \\ 6\sqrt{\frac{3}{35}}V_T & -\frac{12}{\sqrt{35}}V_T & -\frac{6}{\sqrt{35}}V_T & 0 & \frac{12}{7\sqrt{5}}V_T & V_C - \frac{10}{7}V_T \end{pmatrix}, \quad (\text{A23})$$

$$V_{1(2-)}^\pi = \begin{pmatrix} -V_C - \frac{1}{5}V_T & \frac{3\sqrt{6}}{5}V_T & 2V_C - \frac{1}{5}V_T & \frac{3\sqrt{6}}{5}V_T \\ \frac{3\sqrt{6}}{5}V_T & -V_C - \frac{4}{5}V_T & \frac{3\sqrt{6}}{5}V_T & 2V_C - \frac{4}{5}V_T \\ 2V_C - \frac{1}{5}V_T & \frac{3\sqrt{6}}{5}V_T & -V_C - \frac{1}{5}V_T & \frac{3\sqrt{6}}{5}V_T \\ \frac{3\sqrt{6}}{5}V_T & 2V_C - \frac{4}{5}V_T & \frac{3\sqrt{6}}{5}V_T & -V_C - \frac{4}{5}V_T \end{pmatrix}, \quad (\text{A24})$$

The ρ and ω potentials are

$$V_{0(0-)}^v = (2V_C^v - 2V_T^v + V_C^{v'}), \quad (\text{A25})$$

$$V_{0(1+)}^v = \begin{pmatrix} -2V_C^v + V_C^{v'} & -\sqrt{2}V_T^v & 4V_C^v & -\sqrt{2}V_T^v \\ -\sqrt{2}V_T^v & -2V_C^v + V_T^v + V_C^{v'} & -\sqrt{2}V_T^v & 4V_C^v + V_T^v \\ 4V_C^v & -\sqrt{2}V_T^v & -2V_C^v + V_C^{v'} & -\sqrt{2}V_T^v \\ -\sqrt{2}V_T^v & 4V_C^v + V_T^v & -\sqrt{2}V_T^v & -2V_C^v + V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A26})$$

$$V_{0(1-)}^v = \begin{pmatrix} V_C^{v'} & 0 & -2\sqrt{3}V_C^v & 2\sqrt{\frac{3}{5}}V_T^v & -3\sqrt{\frac{2}{5}}V_T^v \\ 0 & 2V_C^v + V_T^v + V_C^{v'} & 0 & 3\sqrt{\frac{3}{5}}V_T^v & -3\sqrt{\frac{2}{5}}V_T^v \\ -2\sqrt{3}V_C^v & 0 & -4V_C^v + V_C^{v'} & -\frac{2}{\sqrt{5}}V_T^v & \sqrt{\frac{6}{5}}V_T^v \\ 2\sqrt{\frac{3}{5}}V_T^v & 3\sqrt{\frac{3}{5}}V_T^v & -\frac{2}{\sqrt{5}}V_T^v & 2V_C^v + \frac{7}{5}V_T^v + V_C^{v'} & -\frac{\sqrt{6}}{5}V_T^v \\ -3\sqrt{\frac{2}{5}}V_T^v & -3\sqrt{\frac{2}{5}}V_T^v & \sqrt{\frac{6}{5}}V_T^v & -\frac{\sqrt{6}}{5}V_T^v & 2V_C^v + \frac{8}{5}V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A27})$$

$$V_{0(2+)}^v = \begin{pmatrix} -2V_C^v - V_T^v + V_C^{v'} & 4V_C^v - V_T^v \\ 4V_C^v - V_T^v & -2V_C^v - V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A28})$$

$$V_{0(2-)}^v = \begin{pmatrix} 2V_C^v - \frac{1}{5}V_T^v + V_C^{v'} & \frac{3\sqrt{6}}{5}V_T^v & -\frac{3\sqrt{3}}{5}V_T^v & \frac{6\sqrt{3}}{5}V_T^v \\ \frac{3\sqrt{6}}{5}V_T^v & 2V_C^v - \frac{4}{5}V_T^v + V_C^{v'} & -\frac{3\sqrt{2}}{5}V_T^v & \frac{6\sqrt{2}}{5}V_T^v \\ -\frac{3\sqrt{3}}{5}V_T^v & -\frac{3\sqrt{2}}{5}V_T^v & 2V_C^v - \frac{7}{5}V_T^v + V_C^{v'} & -\frac{6}{5}V_T^v \\ \frac{6\sqrt{3}}{5}V_T^v & \frac{6\sqrt{2}}{5}V_T^v & -\frac{6}{5}V_T^v & 2V_C^v + \frac{2}{5}V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A29})$$

$$V_{1(0+)}^v = \begin{pmatrix} V_C^{v'} & -2\sqrt{3}V_C^v & -\sqrt{6}V_T^v \\ -2\sqrt{3}V_C^v & -4V_C^v + V_C^{v'} & \sqrt{2}V_T^v \\ -\sqrt{6}V_T^v & \sqrt{2}V_T^v & 2V_C^v + 2V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A30})$$

$$V_{1(0-)}^v = \begin{pmatrix} -2V_C^v + 2V_T^v + V_C^{v'} & 4V_C^v + 2V_T^v \\ 4V_C^v + 2V_T^v & -2V_C^v + 2V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A31})$$

$$V_{1(1+)}^v = \begin{pmatrix} 2V_C^v + V_C^{v'} & \sqrt{2}V_T^v & \sqrt{6}V_T^v \\ \sqrt{2}V_T^v & 2V_C^v - V_T^v + V_C^{v'} & \sqrt{3}V_T^v \\ \sqrt{6}V_T^v & \sqrt{3}V_T^v & 2V_C^v + V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A32})$$

$$V_{1(1-)}^v = \begin{pmatrix} -2V_C^v - V_T^v + V_C^{v'} & -4V_C^v + V_T^v \\ -4V_C^v + V_T^v & -2V_C^v - V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A33})$$

$$V_{1(2+)}^v = \begin{pmatrix} V_C^{v'} & 0 & -2\sqrt{3}V_C^v & -\sqrt{\frac{6}{5}}V_T^v & 2\sqrt{\frac{3}{7}}V_T^v & -6\sqrt{\frac{3}{35}}V_T^v \\ 0 & 2V_C^v + V_T^v + V_C^{v'} & 0 & -3\sqrt{\frac{2}{5}}V_T^v & \frac{3}{\sqrt{7}}V_T^v & \frac{12}{\sqrt{35}}V_T^v \\ -2\sqrt{3}V_C^v & 0 & -4V_C^v + V_C^{v'} & \sqrt{\frac{2}{5}}V_T^v & -\frac{2}{\sqrt{7}}V_T^v & \frac{6}{\sqrt{35}}V_T^v \\ -\sqrt{\frac{6}{5}}V_T^v & 3\sqrt{\frac{3}{5}}V_T^v & \sqrt{\frac{2}{5}}V_T^v & 2V_C^v + V_C^{v'} & -\sqrt{\frac{14}{5}}V_T^v & 0 \\ 2\sqrt{\frac{3}{7}}V_T^v & \frac{3}{\sqrt{7}}V_T^v & -\frac{2}{\sqrt{7}}V_T^v & -\sqrt{\frac{14}{5}}V_T^v & 2V_C^v - \frac{3}{7}V_T^v + V_C^{v'} & -\frac{12}{7\sqrt{5}}V_T^v \\ -6\sqrt{\frac{3}{35}}V_T^v & \frac{12}{\sqrt{35}}V_T^v & \frac{6}{\sqrt{35}}V_T^v & 0 & -\frac{12}{7\sqrt{5}}V_T^v & 2V_C^v + \frac{10}{7}V_T^v + V_C^{v'} \end{pmatrix}, \quad (\text{A34})$$

$$V_{1(2-)}^v = \begin{pmatrix} -2V_C^v + \frac{1}{5}V_T^v + V_C^{v'} & -\frac{3\sqrt{6}}{5}V_T^v & 4V_C^v + \frac{1}{5}V_T^v & -\frac{3\sqrt{6}}{5}V_T^v \\ -\frac{3\sqrt{6}}{5}V_T^v & -2V_C^v + \frac{4}{5}V_T^v + V_C^{v'} & -\frac{3\sqrt{6}}{5}V_T^v & 4V_C^v + \frac{4}{5}V_T^v \\ 4V_C^v + \frac{1}{5}V_T^v & -\frac{3\sqrt{6}}{5}V_T^v & -2V_C^v + \frac{1}{5}V_T^v + V_C^{v'} & -\frac{3\sqrt{6}}{5}V_T^v \\ -\frac{3\sqrt{6}}{5}V_T^v & 4V_C^v + \frac{4}{5}V_T^v & -\frac{3\sqrt{6}}{5}V_T^v & -2V_C^v + \frac{4}{5}V_T^v + V_C^{v'} \end{pmatrix}. \quad (\text{A35})$$

where the central and tensor potentials are defined as,

$$V_C^\pi = \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} C(r; m_\pi) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (\text{A36})$$

$$V_T^\pi = \left(\sqrt{2} \frac{g}{f_\pi} \right)^2 \frac{1}{3} T(r; m_\pi) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (\text{A37})$$

$$V_C^\rho = (2\lambda g_V)^2 \frac{1}{3} C(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (\text{A38})$$

$$V_C^\omega = (2\lambda g_V)^2 \frac{1}{3} C(r; m_\omega), \quad (\text{A39})$$

$$V_T^\rho = (2\lambda g_V)^2 \frac{1}{3} T(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (\text{A40})$$

$$V_T^\omega = (2\lambda g_V)^2 \frac{1}{3} T(r; m_\omega), \quad (\text{A41})$$

$$V_C^{\rho'} = \left(\frac{\beta g_V}{2m_\rho} \right)^2 \frac{1}{3} C(r; m_\rho) \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (\text{A42})$$

$$V_C^{\omega'} = \left(\frac{\beta g_V}{2m_\omega} \right)^2 \frac{1}{3} C(r; m_\omega). \quad (\text{A43})$$

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